A Constitutive Model for Shear Induced Anisotropic Degradation of Weak Sandstone

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ABSTRACT

Weak sandstones possess deformaional behaviors different from hard rocks; these phenomena, including shear dilation and degradation of deformational moduli, are much more significant. Therefore, a model capable of simulating major deformational characteristics of weak sandstones is essentially needed for engineering purposes. An innovative constitutive model is accordingly proposed. The proposed model was formulated based on the linear elastic model, and it accounts for the variations of moduli $K$ and $G$ through different loading conditions. In addition, an anisotropic factor $\beta$ is introduced to reflect the stress-induced anisotropy. It was found that the proposed model is able to closely simulate the actual deformational characteristics of weak sandstones.

The proposed model was then incorporated into a finite element program and was used to analyze a squeezing tunnel case. Overall, this model can describe the deformation behavior for weak sandstones, especially on the significant shear dilation prior to the failure state. As a result, the proposed model shows the versatility in its applicability.

1 Introduction

The western region of Taiwan is most populous and accompanied with active constructions of the transportation infrastructure. Many tunnel constructions currently in progress or in planning are, or to be, constructed in sedimentary strata of Tertiary Period. Due to this relatively young rock-geneses period, weathering and other factors, these sedimentary strata are mostly weak rocks. In the past, these weak rocks have caused several engineering difficulties such as squeezing of the tunnel under construction due to shear-induced deformations [1]. It was found that some typical weak rocks exhibit problematic characteristics such as substantial wet weakening, shear-dilation as well as creep deformation. Such behavior is often much less significant in hard rocks. In order to realize the deformation characteristics of weak sandstone, a series of laboratory tests including pure-shear triaxial tests and creep tests were performed by Jeng et al. [1], Weng et al. [2] and Tsai et al. [3]. According to the results of these researches, weak rocks typically exhibit the following behaviors:

(1) In the hydrostatic loading stage, the total strain possesses nonlinear behavior, which indicates that bulk modulus would increase as hydrostatic stress increases.
(2) In the shear loading stage, the initial shear modulus increases with increasing hydrostatic pressure applied.
(3) The volumetric strain induced by shear is initially contractive, and then gradually transits to be dilative upon increases of shear stresses.

Since squeezing phenomenon in tunnel constructions is inherently related to the aforementioned shear-induced deformation, proper assessments for the rock mass prone to such behavior is of interest in engineering practice. Therefore, it is needed to develop a constitutive model that can properly describe these deformational characteristics.

2 Model formulation

2.1 Model concept

Incorporating the characteristics of deformation behavior of sandstone, especially for shear contraction/dilation, the compliance matrix in the principal stress coordinate is proposed accordingly based on Weng et al. [4] and Graham and Houlsby [5] as:

$$
\begin{bmatrix}
\delta c_1 \\
\delta c_2 \\
\delta c_3 \\
\end{bmatrix} =
\begin{bmatrix}
1/K + 1/3G & 1/K & 1/9K \\
K & 1/K + 1/3G & 1/9K \\
1/9K & 1/K & 1/9K + 1/3G \\
\end{bmatrix}
\begin{bmatrix}
\delta \sigma_1 \\
\delta \sigma_2 \\
\delta \sigma_3 \\
\end{bmatrix}
$$

(1)

where $K$ and $G$ are tangent bulk modulus and shear modulus; $\beta$ is the anisotropic factor; $\delta \sigma_1$, $\delta \sigma_2$, and $\delta \sigma_3$ are the major, intermediate and minor stress increments respectively along the three directions of the principal stresses; and $\delta c_1$, $\delta c_2$, and $\delta c_3$ are the corresponding principal strain increments.

When $\beta$ equals to 1, Eqn. 1 in fact is a constitutive relation of isotropic, linear elastic material. However, sandstone under shearing, for instance, $\beta$ can differ from 1 so that shear-induced volumetric deformation would happen.

Therefore, based on aforementioned observations from experiments, three major features of sandstone are concluded and are to be modeled:
(1) The materials, if they exhibit isotropic nature before shearing, tend to deform isotropically under hydrostatic loadings, with deformation moduli increasing or stiffening upon increasing confining pressure;

(2) The deformation moduli, including $K$ and $G$, may degrade as shear loading increases; and

(3) The direction of “anisotropic” degradation is taken to be the axis of the major principal stress based on the previous research [4]. Moreover, the anisotropic factor $\beta$ is involved to reflect the different degrees of degradation of shear moduli along different directions.

Furthermore, based on Eqn. 1, the volumetric strain increment $\delta\varepsilon_v$, and the shear strain increment $\delta\gamma$ can be expressed in terms of the deformation moduli and the applied stress increments as shown by Eqns. 2 and 3.

$$\delta\varepsilon_v = \frac{1}{3K} \delta I_1 + \left(\frac{1}{9G} + \frac{1}{9GB^2} - \frac{2}{9GB} \right)\delta I_1 + \left(\frac{2}{9G} - \frac{1}{9GB} - \frac{1}{9GB^2}\right)\sqrt{\delta J_2}$$

(2)

$$\delta\gamma = 2\sqrt{\delta J_2} = 2\left(\frac{1}{3G} + \frac{1}{3GB} + \frac{1}{12GB^2}\right)\delta J_2$$

(3)

$$\delta(\varepsilon_v)_{p} = \left(\frac{1}{3K} + \frac{1}{9G} + \frac{1}{9GB^2} - \frac{2}{9GB}\right)\delta I_1$$

(4)

$$\delta(\varepsilon_v)_{s} = \left(\frac{2}{9G} - \frac{1}{9GB^2} - \frac{1}{9GB}\right)\sqrt{\delta J_2}$$

(5)

where $\delta I_1$ is the increment of the first stress invariant; $\delta J_2$ and $\delta J_s$ are the increments of second deviatoric stress and strain invariants, respectively.

Eqn. 5 shows two interesting features of the proposed model: (1) Shearing can induce volumetric deformation; whereas such phenomenon cannot be modeled using isotropic linear elastic models; and (2) Depending on the value of $\beta$, the shear-induced volumetric deformation can be either compressive or dilative. As indicated in Eqn. 5, if $\beta$ is greater than 1, the volumetric strain increment $\delta(\varepsilon_v)_{p}$ will be compressive; if $\beta$ is smaller than 1, the $\delta(\varepsilon_v)_{s}$ will be dilative.

2.2 The variations of moduli and anisotropic factor of sandstones

The next step is to determine the function forms of $K$, $G$, and $\beta$ based on the behaviors of sandstone. The proposed model is designated to be simple both in parameter numbers and in function forms, and it tries to simulate the major degradational behavior of sandstone. As a result, simplified function forms are proposed as follows.

When sandstone is subjected to increasing hydrostatic loading, its stiffness may increase possibly owing to the closer packing of particles inside the material. Therefore, the initial shear modulus $G_0$ is correspondingly set to be linear with the variation of hydrostatic stress as:

$$G_0 = a \times \left(\frac{1}{3} I_1\right) + b$$

(6)

So is the initial bulk modulus $K_0$, and we have:

$$K_0 = c \times \left(\frac{1}{3} I_1\right) + d$$

(7)

where parameters $a$ and $c$ determine how rapid the increase of initial shear and bulk moduli with increasing confining pressure; $b$ and $d$ are the initial values of shear and bulk moduli with no confining pressure. The greater the four values, the stiffer the shear and bulk moduli.

When the material is subjected to shear loading, the shear induces degradation of all deformation moduli. Based on experimental results, the typical degradation of $G$ corresponding to shear stress increasing, and the degradation behavior is often nonlinear during the stages of shearing. As such, the shear modulus variation of $G$ can be expressed in terms of $G_0$ and stress levels, as shown in Eqn. 8:

$$G = G_0 \left(1 - \left(\frac{\sqrt{J_2}}{J_{2,f}}\right)^{\alpha}\right)^{-1}$$

(8)

where $\sqrt{J_{2,f}}$ is the shear strength, which has the form as: $\sqrt{J_{2,f}} = \alpha I_1 + k$. $\alpha$, $k$ are the material parameters. Similar relationships for soils have also been proposed [6, 7].

The degradation of bulk modulus $K$ can also take a similar function form as:

$$K = K_0 \left(1 - \left(\frac{J_2}{J_{2,f}}\right)^{\alpha}\right)$$

(9)

Notably, the application of shear stress will induce anisotropic deformation and the factor $\beta$ will no longer be 1. The variation of $\beta$ under shearing can be described by Eqn. 10 as:

$$\beta = \beta_0 - \frac{\sqrt{J_2}}{J_{2,f}}$$

(10)

The parameter $\beta_0$ reflects the initial degree of anisotropy. When $\beta_0$ is greater than 1, the more $\beta_0$ results in more initial contraction. When $\beta_0$ is less than or equal to 1, monotonic shear dilation can be obtained.
3 Simulations of triaxial tests

To understand the predictive capability of the proposed model in simulating deformational behaviors under different hydrostatic stresses, the deformations at hydrostatic stress of 20 to 80 MPa are simulated and compared. There are seven material parameters ($\alpha, k, a, b, c, d$ and $\beta_0$) to be determined from experimental results. From the triaxial test under hydrostatic pressure of 40 MPa, the corresponding material parameters are obtained and summarized in Table 1.

<table>
<thead>
<tr>
<th>Model</th>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed model</td>
<td>$\alpha$</td>
<td>0.375</td>
</tr>
<tr>
<td></td>
<td>$k$</td>
<td>9.7 MPa</td>
</tr>
<tr>
<td></td>
<td>$a$</td>
<td>52.9</td>
</tr>
<tr>
<td></td>
<td>$b$</td>
<td>8000 MPa</td>
</tr>
<tr>
<td></td>
<td>$c$</td>
<td>196.4</td>
</tr>
<tr>
<td></td>
<td>$d$</td>
<td>1451.3 MPa</td>
</tr>
<tr>
<td></td>
<td>$\beta_0$</td>
<td>1.42</td>
</tr>
</tbody>
</table>

The results of these simulations are to be discussed as bellows. Figure 1a shows the simulated and the measured volumetric strains under pure-shear loading state. It is seen that the simulated volumetric strain exhibits initial compression, and then gradually transits to dilation with increasing shear stress, and the simulated strain is close to the actual curve. The variations of measured volumetric strains under different hydrostatic pressures can be reasonably simulated by the proposed model. Furthermore, the simulated shear strains are presented in Fig. 1b. It is seen that shear-induced shear deformations are also reasonably simulated by the proposed model. As a result, the proposed model is capable of modeling deformational behavior of sandstone under different hydrostatic stresses.

Deducing from the experimental results, the variations of modulus $G$ under shear loading are plotted in Fig 2a. So are the variations of anisotropic factor $\beta$, as illustrated in Fig. 2b. In Fig. 2a, the initial moduli increase as hydrostatic pressure arises, as described by Eqn. 6. Afterwards, as shear stress is applied, all moduli degrade according to Eqn. 8, and considerable degradation occurs when the stress approaches the ultimate strength. In Fig. 2b, the factor $\beta$ at first equals to 1 under the hydrostatic loading. While shear stress is applied, $\beta$ starts to deviate from 1. Initially, $\beta$ is greater than 1, and it reflects the initial compression. As shear stress arises, the value of $\beta$ decreases gradually. Finally, $\beta$ becomes smaller than 1, and the tendency of deformation converts from compression into dilation.
4 Application – case study of a tunnel

After checking the validity of the proposed model, the impact of the behavior of weak sandstone to the deformation of a tunnel subjected to excavation can be accordingly analyzed through numerical analyses. The proposed constitutive model was incorporated into a finite element code ABAQUS and is then used to assess deformation of a tunnel under construction.

A well-known squeezing tunnel in Taiwan is re-analyzed. This tunnel, with a height of 16m, a width of 12.4m, and a thickness of overburden ranging from 20m to 170m, was constructed in the area of the Western Foothill of Taiwan, in which weak sedimentary rocks prevail. The rock mass along the tunnel was mostly the studied sandstone. The drill and blast method was used to excavate this tunnel, and the excavation was done in two stages: top heading excavation and invert excavation. A section of the tunnel, known to have suffered from ground squeezing, is shown in Fig. 3 based on the geometry described by Jeng et al. [1].

![Figure 3: Schematic illustration of the geometry of the studied tunnel.](image)

The deformations of the analysis are found as the follows:

Three constitutive models: (1) the elastic model; (2) the elasto-plastic model with Drucker-Prager yield criterion and (3) the proposed model, are used to analyze the deformation of the studied tunnel. The material parameters and properties are listed in Table 2 based on the studied rock mass.

Table 2: Corresponding parameters for the analyzed tunnel

<table>
<thead>
<tr>
<th>Model</th>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed model</td>
<td>$\alpha$</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td>$k$</td>
<td>82 kPa</td>
</tr>
<tr>
<td></td>
<td>$\beta_0$</td>
<td>1.38</td>
</tr>
<tr>
<td></td>
<td>$E$</td>
<td>200 MPa</td>
</tr>
<tr>
<td></td>
<td>$\nu$</td>
<td>0.21</td>
</tr>
<tr>
<td></td>
<td>$\alpha$</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td>$k$</td>
<td>82 kPa</td>
</tr>
<tr>
<td>Elasto-plastic model</td>
<td>$d$</td>
<td>80MPa</td>
</tr>
</tbody>
</table>

Firstly, Fig. 4 illustrates the deformational modulus distribution of the proposed model, including bulk modulus $K$ and shear modulus $G$, in the cross section after the whole excavation. It could be observed that significant degradations of the two moduli all around the tunnel. These moduli degrade to very low values, and the tunnel invert has the severest degradation.

![Figure 4: Variations of the moduli $K$ and $G$ around the cross section after tunnel excavation.](image)

Following, the influence of these moduli degradation on the tunnel deformation is to be shown as follows.

The inward displacements around the tunnel section, based on the three models, are listed in Fig. 5, in which the angle $\theta$, ranging from 0 to 360°, represents the clockwise direction from the crown. It shows that the major inward displacements based on the elastic and elasto-plastic model occur around the crown. However, according to the proposed model, significant inward displacements develop all over the tunnel section, including the crown, sidewalls and the invert.

What accounts for the significant amount of inward displacements obtained by the proposed model
can be further explored by examining the distributions of volumetric strain. Figures 6 shows the distribution of volumetric deformation around the tunnel obtained by the three models, respectively. As shown in Fig. 6a and 6b, a concentrated dilation zone beneath the invert (the shaded areas) was developed based on the elastic and elasto-plastic model. Consequently, the two models predict greater displacements at the invert than at other locations. However, the dilation zones calculated by the proposed models are almost around the whole section and are much larger than that calculated by the elasto-plastic model, as shown in Figs. 6c. As a result, more dilation pushes the crown further inward, as revealed by the proposed model.

5 Conclusions

An anisotropic degradation model is proposed to represent the key deformational characteristics of weak sandstone.

The stress-strain relationship of the proposed model was originated from the degradation of moduli $K$ and $G$ subjected to different loading conditions, and an anisotropic factor $\beta$ is introduced to reflect the tendency shear-induced volumetric deformation. As a result, this proposed constitutive model is characterized by the following features: (1) being capable of describing shear-induced volumetric deformation, either compression or dilation, prior to the failure state; (2) the anisotropic factor $\beta$ provides a clear index on whether shear-induced volumetric deformation dilate or not.

The proposed model has been verified by comparing to experimental results. As a result, it is found that the proposed model is versatile in simulating the deformations of sandstone under different stress paths. Moreover, the model has been incorporated into the finite element program and used to analyze a squeezing tunnel case in Taiwan. When compared with other models, the predictions of the proposed model are closer to reality and indicate a larger crown settlement, owing to larger extent of dilation zones. Overall, the proposed model seems to be able to reasonably describe the deformation behavior weak sandstones.

Reference


