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The Economic Society of Australia warmly welcomes you to the Gold Coast, Queensland, Australia for the 37th Australian Conference of Economists.

The Society was formed 83 years ago in 1925. At the time, the Society was opposed to declarations of policy and instead focused on open discussions and encouraging economic debate. Nothing has changed today, with the Society and the conference being at the forefront of encouraging debate.

This year we have a large number of papers dealing with Infrastructure, Central Banking and Trade.

Matters of the greatest global importance invariably boil down to be economic problems. Recent times have seen an explosion of infrastructure spending, after world-wide population growth has seen demand outpace aging supply. The world has become more globalised than at any time since World War I but the benefits of this (and the impact on our climate) has been questioned by some.

At the time of preparing for this conference we could not have known that it would have been held during the largest credit crisis since the Great Depression. The general public and politicians both look to central banks for the answers.

We are also very pleased to see a wide selection of papers ranging from applied economics to welfare economics. An A – Z of economics (well, almost).

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R&D, Strategic Delegation and Market Share Competition

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Abstract

This paper investigates the role of spillovers in determining manager's decisions of R&D investment and the owner's choices of incentives between a profit-market share framework. We consider a three-stage game in which two firms with homogeneous products pursue oligopolistic competition in the market and each owner delegates the production and R&D decisions to a manager. Each manager's objective is guided by an incentive scheme which is a weighted sum of profits and market share. It is found that with higher R&D spillovers both firms commit more competition since rival firms also benefit from cost-reducing R&D spillovers, therefore they invest less in R&D. In this case, the owner may choose higher weights for market share under certain condition. The cost of investment in R&D also affects the weight on market share. When R&D is relatively costly, the owner may choose lower weights for market share.

Keywords: Managerial delegation; R&D spillovers; Oligopoly; Market share.
JEL-Classification: D21; D43; C72; L13.
1. Introduction

In large companies ownership and management are commonly separated, which may lead to additionally problems that are well known in managerial incentive literature (see, e.g., Jensen and Meckling, 1976; Demsetz, 1983; Fama and Jensen, 1983). In these studies, the owner and manager relationship is treated as a conventional principal-agent issue in which the compensation scheme is proposed to solve manager’s incentive problem. However, as argued in the strategic delegation literature, the owner-manager relationship can be described as a delegation problem in which owners delegate the production and other managerial decisions to managers. Thus, for instance, the managerial contracts considering for sales and market share may direct managers to allow for non-profit maximization (see, Lackman and Craycroft, 1974; Vickers, 1985; Fershtman and Judd, 1987; Sklivas, 1987; Goering, 1996).

Zhang and Zhang (1997) combine the strategic delegation in Fershtman-Judd-Sklivas framework with the aspect of R&D spillovers, adopting the cost-reducing R&D model of d’Aspremont and Jacquemin (1988). By doing this, they discuss how the ownership structure affects firm’s R&D and production decisions, and examine the social welfare effects under two different ownership structures.

This paper investigates the role of spillovers in influencing owner’s incentive schemes and manager’s R&D choices. Using the framework developed by Fershtman and Judd (1987) and Skilvas (1985), we consider that owner links to market share of the incentive scheme, that manager decides whether or not engaging in cost-reducing R&D to increase the market share, in the presence of spillovers. We are able to highlight the role of R&D spillovers in a strategic delegation framework with market share competition a la Zhang and Zhang (1997) and Jansen et al (2007).

The rest of paper is organized as follows. In Section two, the three-stage duopolistic model with strategic delegation and cost-reducing R&D is described. The
third section presents the results of the model. The final section provides concluding remarks.

2. The Model

We consider a three-stage model in which two firms with homogeneous products pursue oligopolistic competition in the market. We assume each firm has a risk neutral owner and a risk neutral manager. In the first stage, two owners simultaneously choose the incentive schemes for their managers, which is a linear combination of profit and market share. In the second stage, the two managers simultaneously make their cost-reducing R&D decisions noncooperatively given the incentive schemes. At the third stage, the two managers simultaneously make their output decisions in a Cournot-Nash game given the incentive schemes and R&D investment of both firms.

In the first stage, owner \( i (i = 1,2) \) chooses incentive scheme \( \alpha_i \) to maximize a linear combination of profit \( (\pi_i) \) and market share \( (s_i) \):

\[
V_i = \pi_i + \alpha_i s_i , \tag{1}
\]

where \( s_i = q_i / Q \), \( s_i'(Q) > 0 \) and \( s_i''(Q) < 0 \), and \( Q \) denotes the industry output, \( Q = q_1 + q_2 \). Each owner can only proposes its incentive scheme, but not determine the R&D and sales. Here, unlike Fershtman and Judd (1987) and Skilvas (1985), we particularly consider the case where the exerting effort to win a higher market share is considered in the managerial compensation contract. By doing so, owner \( i \) uses market share \( s_i \) as a strategic instrument to influence the performance of each manager.

In the second stage, manager \( i \) decides whether or not to engage in cost-reducing R&D activity. Following d’Aspremont and Jaquemin (1988), we assume manager \( i \) chooses R&D investment \( x_i \) to maximize \( V_i \). If both managers \( i \) and \( j \) undertake the
cost-reducing R&D activity, then the post-innovation marginal cost of production is given by \( c_i = k - x_i - \beta x_j \), where \( x_i \) and \( x_j \) are the sizes of R&D cost-reducing by firms \( i \) and \( j \), respectively. \( x_i + \beta x_j \) is the total amount of marginal cost reduction for firm \( i \), where \( k > x_i - \beta x_j \) to assure positive marginal cost. \( \beta \) is used to measure the size of R&D spillover effect between two firms, \( \beta \in [0, 1] \). Thus, a large value of \( \beta \) implies that a firm’s R&D investment has a greater cost reduction on the other’s marginal production cost.

And in the final stage, given the incentive schemes \( \alpha_i \) and \( \alpha_j \) and effective production costs \( c_i \) and \( c_j \) the managers \( i \) and \( j \) choose the quantity and compete for market shares in the product market. Manager \( i \) chooses output \( q_i \) to maximize \( V_i \):

\[
V_i = (P - c_i)q_i - X_i + \hat{a}_i q_i / Q; \quad i = 1,2,
\]

where \( P \) denotes price, \( P = 1 - Q \), \( X_i \) is firm \( i \)’s R&D cost, and \( 0 < k < 1 \). Note that \( P_i'(Q) < 0 \) and \( P_i'(Q) + P_i''(Q)q_i < 0 \). In the following, we solve for a subgame perfect equilibrium to characterize the strategic delegation game with R&D spillovers.

3. Results

We employ the method of backward induction to solve for the three-stage game. In the stage of market competition, the outputs in the Cournot-Nash equilibrium, denoted by \( (q_i^*(c_1,c_2), q_2^*(c_1,c_2)) \), must satisfy the following system of equations:

\[
\begin{align*}
    f(q_1, q_2; c_1, c_2) & \frac{\partial V_1}{\partial q_1} = P'(Q)q_1 + P(Q) - c_1 + \hat{a}_1 s_{11}(Q), \\
    g(q_1, q_2; c_1, c_2) & \frac{\partial V_2}{\partial q_2} = P'(Q)q_2 + P(Q) - c_2 + \hat{a}_2 s_{22}(Q).
\end{align*}
\]
Equation (3) characterizes the best output response function of firm 1 and 2, respectively, at the market stage. It is evident that $f(\cdot)$ and $g(\cdot)$ have at most one solution for any $q_i$ since $\partial f/\partial q_i < 0$ and $\partial g/\partial q_i < 0$. The details are shown in Appendix A.

Using Implicit Function rule, that is,

$$
\begin{align*}
&df(q_1, q_2; c_1, c_2) = f_1 dq_1 + f_2 dq_2 + f_3 dc_1 + f_4 dc_2 = 0, \\
&dg(q_1, q_2; c_1, c_2) = g_1 dq_1 + g_2 dq_2 + g_3 dc_1 + g_4 dc_2 = 0,
\end{align*}
$$

(4)

thus, we have:

$$
\begin{bmatrix}
  f_1 & f_2 \\
  g_1 & g_2
\end{bmatrix}
\begin{bmatrix}
  \frac{\partial q_1}{\partial c_1} \\
  \frac{\partial q_2}{\partial c_1}
\end{bmatrix}
= \begin{bmatrix}
  1 \\
  0
\end{bmatrix}.
$$

Therefore, 

$$
\frac{\partial q_1}{\partial c_1} = \frac{(P^*(Q)q_2 + P'(Q) + \alpha_2 s_{21}(Q))}{\Delta(> 0)} < 0 \text{ if } q_1 < q_2, \text{ and }
$$

$$
\frac{\partial q_2}{\partial c_1} = \frac{-(P^*(Q)q_2 + 2P'(Q) + \alpha_2 s_{22}(Q))}{\Delta(> 0)} > 0.
$$

That is, a decrease (increases) in firm 1’s marginal cost of production increases (decreases) firm 1’s output and decreases (increases) firm 2’s output. This implies that an increase in cost-reducing R&D by firm 1 may increase firm 1’s equilibrium output, by contrast, reduce firm 2’s output.

Next, we solve the manager’s problem for the R&D stage of the game for further discussion.

In the R&D stage, the managers decide the size of cost-reducing R&D investment. Assume firm $i$’s R&D cost function $X_i$ is convex in R&D investment $x_i$, e.g. $X_i = 0.5x_i^2$. Then, using $q_1^*(c_1(x, k, \beta, \gamma), c_2(x, k, \beta, \gamma))$ and $q_2^*(c_1(x, k, \beta, \gamma), c_2(x, k, \beta, \gamma))$, we solve for $x^*(k, \beta, \gamma, \alpha_1, \alpha_2)$. Differentiating equation (2) with respect to $x$ leads to:

$$
\frac{\partial V_i}{\partial x} = \frac{\partial V_i}{\partial q_1} \frac{\partial q_1}{\partial x} + \frac{\partial V_i}{\partial q_2} \frac{\partial q_2}{\partial x} + \frac{\partial V_i}{\partial X_i} \frac{\partial X_i}{\partial x},
$$

(5)

1 Jansen et al (2007) investigate a concave reaction function with identical marginal cost between two competing firms.
By using the first equation in (3), all the expressions involved in equation (5) can be rearranged as:

\[
\frac{\partial V_1}{\partial x} = q_1 \left\{ \left[ P'(Q) + \frac{\alpha_1 s_{12}}{q_1} \right] \frac{\partial q_2}{\partial c_1} \frac{\partial c_1}{\partial x} - 1 \right\} + \left[ P'(Q) + \frac{\alpha_1 s_{12}}{q_1} \right] \frac{\partial q_2}{\partial c_2} \frac{\partial c_2}{\partial x} - \frac{\partial X_1}{\partial x}. \tag{6}
\]

The detailed calculation of equation (6) is shown in Appendix B. Next, using Implicit Function rule, we have

\[
dh(x; k, \beta, \gamma, \alpha_1, \alpha_2) = h_1 dx + h_2 dk + h_3 d\beta + h_4 d\gamma + h_5 d\alpha_1 + h_6 d\alpha_2 = 0, \tag{7}
\]

where

\[
h_1 \int \frac{\partial^2 V_1}{\partial x^2} < 0, \quad h_2 \int \frac{\partial^2 V_1}{\partial \beta \partial k} < 0, \quad h_3 \int \frac{\partial^2 V_1}{\partial x \partial \beta} = q_1 \left\{ P'(Q) + \frac{\alpha_1 s_{12}}{q_1} \right\} \frac{\partial^2 c_1}{\partial x \partial \beta} < 0, \quad h_4 \int \frac{\partial^2 V_1}{\partial x \partial \gamma} = q_1 \left\{ P'(Q) + \frac{\alpha_1 s_{12}}{q_1} \right\} \frac{\partial^2 c_1}{\partial x \partial \gamma} < 0, \quad h_5 \int \frac{\partial^2 V_1}{\partial x \partial \alpha_1} = q_1 \left\{ P'(Q) + \frac{s_{12}}{q_1} \right\} \frac{\partial^2 c_1}{\partial c_1 \partial x} \left[ \frac{\partial c_1}{\partial x} > 0 \right] + \frac{\partial^2 c_2}{\partial c_2 \partial x} \left[ \frac{\partial c_2}{\partial x} > 0 \right] > 0 \quad \text{if} \quad \frac{\dot{\alpha}_2 s_{22}}{\dot{\alpha}_1 s_{11}} < \beta.
\]

As we see, \( \frac{\partial x}{\partial \beta} = -\frac{h_3}{h_1} < 0 \). This implies that with higher R&D spillovers both firms commit more competition since rival firms also benefit from cost-reducing R&D investment through spillovers, therefore they are less willing to investment in cost-reducing R&D. \( \frac{\partial x}{\partial \gamma} = -\frac{h_4}{h_1} < 0 \), indicating that firms are unwilling to when R&D is relatively costly. In addition, \( \frac{\partial x}{\partial \alpha_1} = -\frac{h_5}{h_1} > 0 \) if \( \frac{\dot{\alpha}_2 s_{22}}{\dot{\alpha}_1 s_{11}} < \beta \), i.e. the incentive scheme, proposed by firm 1’s owner for its manager, puts a relatively higher weight
on market share, firm 1’s manager will invest more in R&D.

In the first stage, the two owners simultaneously choose the incentive schemes for their managers. Using \( x^*(\beta, \gamma, k, \alpha_1, \alpha_2) \), we solve for \( \alpha_i^*(\beta, \gamma, k) \) and \( \alpha_2^*(\beta, \gamma, k) \). The optimal values of the incentive schemes can be determined by solving the following system of first-order conditions (see Appendix C for the detailed calculation):

\[
\frac{\partial V_1}{\partial \alpha_1} = \frac{\partial x}{\partial \alpha_1} \left\{ q_1 \left[ \left( \frac{p'(Q)}{q_1} + \frac{\alpha_1 s_{12}}{q_1} \right) \frac{\partial q_2}{\partial c_1} - 1 \right] \frac{\partial c_1}{\partial x} + q_1 \left[ \left( \frac{p'(Q)}{q_1} + \frac{\alpha_1 s_{12}}{q_1} \right) \frac{\partial q_2}{\partial c_2} \right] \frac{\partial c_2}{\partial x} - x \right\} + s_1(Q),
\]

\[
\frac{\partial V_2}{\partial \alpha_2} = \frac{\partial x}{\partial \alpha_2} \left\{ q_2 \left[ \left( \frac{p'(Q)}{q_2} + \frac{\alpha_2 s_{21}}{q_2} \right) \frac{\partial q_1}{\partial c_1} - 1 \right] \frac{\partial c_1}{\partial x} + q_2 \left[ \left( \frac{p'(Q)}{q_2} + \frac{\alpha_2 s_{21}}{q_2} \right) \frac{\partial q_1}{\partial c_2} \right] \frac{\partial c_2}{\partial x} - x \right\} + s_2(Q).
\]

Applying Implicit Function rule to the above equations, we can write:

\[
dm(\alpha_1, \alpha_2; k, \beta, \gamma) \frac{\partial \alpha_1}{\partial m} + m_1 d\alpha_1 + m_2 d\alpha_2 + m_3 dk + m_4 d\beta + m_5 d\gamma = 0,
\]

\[
dn(\alpha_1, \alpha_2; k, \beta, \gamma) \frac{\partial \alpha_2}{\partial n} + n_1 d\alpha_1 + n_2 d\alpha_2 + n_3 dk + n_4 d\beta + n_5 d\gamma = 0,
\]

where

\[
m_1 \frac{\partial^2 V_1}{\partial \alpha_1^2} < 0, \quad n_1 \frac{\partial^2 V_1}{\partial \alpha_2^2} < 0, \quad m_2 \frac{\partial^2 V_1}{\partial \alpha_1 \partial \alpha_2} < 0, \quad n_2 \frac{\partial^2 V_2}{\partial \alpha_2 \partial \alpha_1} < 0,
\]

\[
m_3 \frac{\partial^2 V_1}{\partial \alpha_1 \partial \beta} < 0, \quad n_3 \frac{\partial^2 V_2}{\partial \alpha_2 \partial \beta} < 0, \quad m_4 \frac{\partial^2 V_1}{\partial \alpha_1 \partial \gamma} < 0, \quad n_4 \frac{\partial^2 V_2}{\partial \alpha_2 \partial \gamma} < 0, \quad m_5 \frac{\partial^2 V_1}{\partial \alpha_1 \partial \gamma} < 0, \quad n_5 \frac{\partial^2 V_2}{\partial \alpha_2 \partial \gamma} < 0.
\]

As the determinant of the system of first-order conditions is strictly positive,

\[
\Delta_{s1} = m_1 n_2 - n_1 m_2 > 0,
\]

using the Implicit Function Theorem we have:

\[
\frac{\partial \alpha_1}{\partial \beta} = (n, m_2 - m_1 n_2)/\Delta_{s1}, \quad \frac{\partial \alpha_2}{\partial \beta} = (n, m_3 - m_1 n_2)/\Delta_{s1}, \quad \frac{\partial \alpha_1}{\partial \gamma} = (n, m_2 - m_1 n_2)/\Delta_{s1}, \quad \frac{\partial \alpha_2}{\partial \gamma} = (n, m_3 - m_1 n_2)/\Delta_{s1}.
\]

As shown in the second stage, with higher R&D spillovers both firms commit more competition since rival firms also benefit from cost-reducing R&D spillovers, therefore they invest less in R&D. In this case, the owner may choose higher weights...
for market share if \((n_im_2 - m_in_2) > 0\). Similarly, the owner of rival firm choose higher weights for market share if \((n_im_4 - m_in_4) > 0\). The cost of investment in R&D also affects the weight on market share. When R&D is relatively costly, the owner may choose lower weights for market share if \(n_im_2 < m_in_2\).

4. Conclusion

In this paper, we explore strategic delegation game with market share in the presence of spillovers. Specifically, we reconsider the three-stage model of Zhang and Zhang (1997), but adopt the managerial incentive scheme of Jansen et al (2007). The results show that with higher R&D spillovers both firms commit more competition since rival firms also benefit from cost-reducing R&D spillovers, therefore they invest less in R&D. In this case, the owner may choose higher weights for market share under certain condition. The cost of investment in R&D also affects the weight on market share. When R&D is relatively costly, the owner may choose lower weights for market share

The managerial contracts considering for sales and market share may direct managers to allow for non-profit maximization. In the literature on the managerial incentive schemes, Zhang and Zhang (1997) pioneered in combining the strategic delegation in Fershtman-Judd-Sklivas framework with the aspect of cost-reducing R&D spillovers. It is well known that in the game of strategic delegation with R&D spillovers there are two counter effects. Strategic delegation may direct managers to act more aggressively. On the other hand, R&D spillovers reduce incentives to act aggressively since each firm can also benefit from its rival’s cost-reducing R&D.

The results obtained in this paper suggest that industry-specific primary cost and its interaction with the level of spillovers determine the manager's R&D choice. Further, the choice of R&D investment need not be influenced by the degree of
spillovers when the owner's incentive scheme is market share based. This result offers an alternative explanation for the selected negotiations over intellectual property rights infringement between the developed countries and their developing counterparts. Our model can be extended to the study of trade policy in the light of cross-border intellectual property violations. Indeed, if owner’s choice in the first stage is replaced by government decisions over policy measures, such as tariff or subsidy, the current model can easily project into analyses of trade-related intellectual property protection.
Appendix A.

The outputs \((q^*_1(c_1, c_2), q^*_2(c_1, c_2))\) in the Cournot-Nash equilibrium satisfy the equations in (3). Then,

\[
\begin{align*}
    f_1(q_1, q_2; c_1, c_2) \int \frac{\partial^2 V}{\partial q_1^2} &= P'(Q)q_1 + 2P'(Q) + \alpha_1 s_{11}(Q), \\
    f_2(q_1, q_2; c_1, c_2) \int \frac{\partial^2 V}{\partial q_1 \partial q_2} &= P'(Q)q_1 + P'(Q) + \alpha_1 s_{12}(Q), \\
    f_3(q_1, q_2; c_1, c_2) \int \frac{\partial^2 V}{\partial q_2^2} &= -1, \\
    f_4(q_1, q_2; c_1, c_2) \int \frac{\partial^2 V}{\partial q_2 \partial c_1} &= 0.
\end{align*}
\]

Appendix B.

\[
\begin{align*}
    \frac{\partial V_1}{\partial x} &= \left( \frac{\partial P(Q)}{\partial x} - \frac{\partial c_1}{\partial x} \right) q_1 + \left( P(Q) - c_1 \right) \frac{\partial q_1}{\partial x} + \alpha_1 \left( s_{11} \frac{\partial q_1}{\partial x} + s_{12} \frac{\partial q_2}{\partial x} \right) - \frac{\partial X_1}{\partial x} \\
    &= \left( \frac{\partial P(Q)}{\partial q_1} \frac{\partial q_1}{\partial x} \frac{\partial P(Q)}{\partial q_2} \frac{\partial q_2}{\partial x} - \frac{\partial c_1}{\partial x} \right) q_1 + \left( P(Q) - c_1 + \alpha_1 s_{11} \right) \frac{\partial q_1}{\partial x} + \alpha_1 s_{12} \frac{\partial q_2}{\partial x} - \frac{\partial X_1}{\partial x} \\
    &= -P(Q)q_1 \\
    &= \left( \frac{\partial^2 V}{\partial q_1^2} \right) q_1 + \alpha_1 s_{12} \frac{\partial q_2}{\partial q_1} - \frac{\partial X_1}{\partial x} \\
    &= q_1 \left( \left[ \frac{\partial P(Q)}{\partial q_2} + \alpha_1 s_{12} \right] \frac{\partial q_2}{\partial q_1} - 1 \right) \frac{\partial c_1}{\partial x} + \left[ \frac{\partial P(Q)}{\partial q_2} + \alpha_1 s_{12} \right] \frac{\partial q_2}{\partial q_1} \frac{\partial c_2}{\partial x} - \frac{\partial X_1}{\partial x}
\end{align*}
\]
Appendix C.

\[
\frac{\partial V_1}{\partial \alpha_1} = \left(P'(Q) - \frac{\partial c_1}{\partial \alpha_1}\right) q_1 + \left(P(Q) - c_1 + \alpha_1 s_{1 i}\right) \frac{\partial q_1}{\partial \alpha_1} + \left(s_1(Q) + \alpha_1 s_{1 2} \frac{\partial q_2}{\partial \alpha_1}\right) - \frac{\partial X_1}{\partial \alpha_1}
\]

\[
= q_1 \left(\frac{\partial P(Q)}{\partial q_1} \frac{\partial q_1}{\partial \alpha_1} + \frac{\partial P(Q)}{\partial q_2} \frac{\partial q_2}{\partial \alpha_2} - \frac{\partial c_1}{\partial \alpha_1}\right) - P'(Q) q_1 \frac{\partial q_1}{\partial \alpha_1} + \left(s_1(Q) + \alpha_1 s_{1 2} \frac{\partial q_2}{\partial \alpha_1}\right) - \frac{\partial X_1}{\partial \alpha_1}
\]

\[
= (P'(Q) q_1 + \alpha_1 s_{1 2}) \frac{\partial q_2}{\partial \alpha_1} - q_1 \frac{\partial c_1}{\partial \alpha_1} - \frac{\partial X_1}{\partial \alpha_1} + s_1(Q)
\]

\[
= (P'(Q) q_1 + \alpha_1 s_{1 2}) \left[\frac{\partial q_2}{\partial c_1} \frac{\partial c_1}{\partial x} + \frac{\partial q_2}{\partial c_2} \frac{\partial c_2}{\partial x}\right] - q_1 \frac{\partial c_1}{\partial x} - \frac{\partial X_1}{\partial x} + s_1(Q)
\]

\[
= \frac{\partial x}{\partial \alpha_1} \left\{P'(Q) q_1 + \alpha_1 s_{1 2}\right\} \left[\frac{\partial q_2}{\partial c_1} \frac{\partial c_1}{\partial x} + \frac{\partial q_2}{\partial c_2} \frac{\partial c_2}{\partial x}\right] - q_1 \frac{\partial c_1}{\partial x} - \frac{\partial x}{\partial \alpha_1} + s_1(Q)
\]

\[
\frac{\partial V_2}{\partial \alpha_2} = (P'(Q) q_2 + \alpha_2 s_{2 1}) \frac{\partial q_1}{\partial \alpha_2} - q_2 \frac{\partial c_2}{\partial \alpha_2} - \frac{\partial X_2}{\partial \alpha_2} + s_2(Q)
\]

\[
= (P'(Q) q_2 + \alpha_2 s_{2 1}) \left[\frac{\partial q_1}{\partial c_1} \frac{\partial c_1}{\partial x} + \frac{\partial q_1}{\partial c_2} \frac{\partial c_2}{\partial x}\right] - q_2 \frac{\partial c_2}{\partial x} - \frac{\partial X_2}{\partial x} + s_2(Q)
\]

\[
= \frac{\partial x}{\partial \alpha_2} \left\{P'(Q) q_2 + \alpha_2 s_{2 1}\right\} \left[\frac{\partial q_1}{\partial c_1} \frac{\partial c_1}{\partial x} + \frac{\partial q_1}{\partial c_2} \frac{\partial c_2}{\partial x}\right] - q_2 \frac{\partial c_2}{\partial x} - \frac{\partial x}{\partial \alpha_2} + s_2(Q)
\]

\[
= \frac{\partial x}{\partial \alpha_2} \left\{q_2 \left[P'(Q) + \frac{\alpha_2 s_{2 1}}{q_2} \frac{\partial q_1}{\partial c_1} \frac{\partial c_1}{\partial x} + \frac{\partial q_1}{\partial c_2} \frac{\partial c_2}{\partial x}\right] - \frac{\partial x}{\partial \alpha_2} + s_2(Q)
\]

Reference


