普通物理

Lecture 13 & 14
Solids & Fluids
固體與流體
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State of Matter

The Solid

- Has definite **volume**
- Has definite **shape**
- Molecules are held in specific locations
  - by **electrical forces**
- Vibrate about equilibrium positions
- Can be modeled as springs connecting molecules
External forces can be applied to the solid and compress the material
   – In the model, the springs would be compressed

When the force is removed, the solid returns to its original shape and size
   – This property is called *elasticity*
State of Matter 物質的型態

Crystalline Solid
- Atoms have an ordered structure
- This example is salt
  - Gray spheres represent Na$^+$ ions
  - Green spheres represent Cl$^-$ ions

Amorphous Solid
- Atoms are arranged almost randomly
- Examples include glass
State of Matter

The Liquid

- Has a definite volume
- No definite shape
- Exists at a higher temperature than solids
- The molecules “wander” through the liquid in a random fashion
  - The intermolecular forces are not strong enough to keep the molecules in a fixed position
State of Matter

The Gas

- Has no definite volume
- Has no definite shape
- Molecules are in constant random motion
- The molecules exert only weak forces on each other
- Average distance between molecules is larger compared to the size of the molecules
State of Matter 物質的型態

The Plasma 游離氣體・電漿・等離子體

- Matter heated to a very high temperature
- Many of the electrons are freed from the nucleus
- Result is a collection of free, electrically charged ions
- Plasmas exist inside stars
Deformation of Solids

- All objects are deformable
- It is possible to change the shape or size (or both) of an object through the application of external forces
- When the forces are removed, the object tends to its original shape
  - This is a deformation that exhibits *elastic behavior*
Deformation of Solids

Elastic Properties

- **Stress** is the force per unit area causing the deformation
- **Strain** is a measure of the amount of deformation
- The *elastic modulus* is the constant of proportionality between stress and strain
  - For sufficiently small stresses, the stress is directly proportional to the strain
  - The constant of proportionality depends on the material being deformed and the nature of the deformation
Deformation of Solids

Elastic Modulus

- The elastic modulus can be thought of as the **stiffness** of the material
  - A material with a large elastic modulus is very stiff and difficult to deform
    - Analogous to the spring constant
  - \( \text{Stress} = \text{Elastic Modulus} \times \text{Strain} \)
Deformation of Solids

Young’s Modulus: Elasticity in Length

- Tensile stress is the ratio of the external force to the cross-sectional area
  - Tensile is because the bar is under tension
- The elastic modulus is called **Young’s modulus**
Deformation of Solids

- SI units of stress are Pascals, Pa
  - 1 Pa = 1 N/m²

- The tensile strain is the ratio of the change in length to the original length
  - Strain is dimensionless

- Young’s modulus applies to a stress of either tension or compression

- It is possible to exceed the elastic limit of the material
  - No longer directly proportional
  - Ordinarily does not return to its original length
Deformation of Solids

Breaking

- If stress continues, it surpasses its ultimate strength
  - The ultimate strength is the greatest stress the object can withstand without breaking
- The breaking point
  - For a brittle material, the breaking point is just beyond its ultimate strength
  - For a ductile material, after passing the ultimate strength the material thins and stretches at a lower stress level before breaking
Deformation of Solids

Shear Modulus: Elasticity of Shape

- Forces may be parallel to one of the object’s faces
- The stress is called a shear stress
- The shear strain is the ratio of the horizontal displacement and the height of the object
- The shear modulus is S
Deformation of Solids

- Shear stress: \[ \tau = \frac{F}{A} \]
- Shear strain: \[ \gamma = \frac{\Delta x}{h} \]

\[ \frac{F}{A} = S \frac{\Delta x}{h} \]

- S is the shear modulus
- A material having a large shear modulus is difficult to bend
Deformation of Solids

Bulk Modulus: Volume Elasticity

- Bulk modulus characterizes the response of an object to **uniform squeezing**
  - Suppose the forces are perpendicular to, and act on, all the surfaces
    - Example: when an object is immersed in a fluid
- The object undergoes a change in volume without a change in shape
Deformation of Solids

- Volume stress, $\Delta P$, is the ratio of the force to the surface area
  - This is also the Pressure

- The volume strain is equal to the ratio of the change in volume to the original volume

$$\Delta P = -B \frac{\Delta V}{V}$$
Deformation of Solids

- A material with a large bulk modulus is difficult to compress
- The negative sign is included since an increase in pressure will produce a decrease in volume
  - \( B \) is always positive
- The \textit{compressibility} is the reciprocal of the bulk modulus

\[
C = \frac{1}{B}
\]
Deformation of Solids

- **Solids** have Young’s, Bulk, and Shear moduli
- **Liquids** have only bulk moduli, they will not undergo a shearing or tensile stress
  - The liquid would flow instead
# Deformation of Solids

## Typical Values for the Elastic Modulus

<table>
<thead>
<tr>
<th>Substance</th>
<th>Young's Modulus (Pa)</th>
<th>Shear Modulus (Pa)</th>
<th>Bulk Modulus (Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>$7.0 \times 10^{10}$</td>
<td>$2.5 \times 10^{10}$</td>
<td>$7.0 \times 10^{10}$</td>
</tr>
<tr>
<td>Bone</td>
<td>$1.8 \times 10^{10}$</td>
<td>$8.0 \times 10^{10}$</td>
<td>—</td>
</tr>
<tr>
<td>Brass</td>
<td>$9.1 \times 10^{10}$</td>
<td>$3.5 \times 10^{10}$</td>
<td>$6.1 \times 10^{10}$</td>
</tr>
<tr>
<td>Copper</td>
<td>$11 \times 10^{10}$</td>
<td>$4.2 \times 10^{10}$</td>
<td>$14 \times 10^{10}$</td>
</tr>
<tr>
<td>Steel</td>
<td>$20 \times 10^{10}$</td>
<td>$8.4 \times 10^{10}$</td>
<td>$16 \times 10^{10}$</td>
</tr>
<tr>
<td>Tungsten</td>
<td>$35 \times 10^{10}$</td>
<td>$14 \times 10^{10}$</td>
<td>$20 \times 10^{10}$</td>
</tr>
<tr>
<td>Glass</td>
<td>$6.5 - 7.8 \times 10^{10}$</td>
<td>$2.6 - 3.2 \times 10^{10}$</td>
<td>$5.0 - 5.5 \times 10^{10}$</td>
</tr>
<tr>
<td>Quartz</td>
<td>$5.6 \times 10^{10}$</td>
<td>$2.6 \times 10^{10}$</td>
<td>$2.7 \times 10^{10}$</td>
</tr>
<tr>
<td>Rib Cartilage</td>
<td>$1.2 \times 10^{7}$</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Rubber</td>
<td>$0.1 \times 10^{7}$</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Tendon</td>
<td>$2 \times 10^{7}$</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Water</td>
<td>—</td>
<td>—</td>
<td>$0.21 \times 10^{10}$</td>
</tr>
<tr>
<td>Mercury</td>
<td>—</td>
<td>—</td>
<td>$2.8 \times 10^{10}$</td>
</tr>
</tbody>
</table>
Ultimate Strength of Materials

- The *ultimate strength* of a material is the maximum force per unit area the material can withstand before it breaks or factures.

- Some materials are stronger in compression than in tension.
Example 1

Problem  A vertical steel beam in a building supports a load of $6.0 \times 10^4$ N. (a) If the length of the beam is 4.0 m and its cross-sectional area is $8.0 \times 10^{-3}$ m$^2$, find the distance the beam is compressed along its length. (b) What maximum load in newtons could the steel beam support before failing?  

Assuming ultimate strength=$5\times10^8$ kPa

\[
\frac{F}{A} = \frac{F}{Y} \frac{\Delta L}{L_0} \\
\Delta L = \frac{FL_0}{YA} = \frac{(6.0 \times 10^4 \text{ N})(4.0 \text{ m})}{(2.0 \times 10^{11} \text{ Pa})(8.0 \times 10^{-3} \text{ m}^2)} \\
= 1.5 \times 10^{-4} \text{ m}
\]

\[
\frac{F}{A} = \frac{F}{8.0 \times 10^{-3} \text{ m}^2} = 5.0 \times 10^8 \text{ Pa} \\
F = 4.0 \times 10^6 \text{ N}
\]
Example 2

Problem  A solid lead sphere of volume 0.50 m³, dropped in the ocean, sinks to a depth of 2.0 \times 10³ m (about 1 mile), where the pressure increases by 2.0 \times 10⁷ Pa. Lead has a bulk modulus of 4.2 \times 10¹⁰ Pa. What is the change in volume of the sphere?

\[
B = -\frac{\Delta P}{\Delta V/V} \\
\Delta V = -\frac{V\Delta P}{B} \\
\Delta V = -\frac{(0.50 \text{ m}^3)(2.0 \times 10^7 \text{ Pa})}{4.2 \times 10^{10} \text{ Pa}} = -2.4 \times 10^{-4} \text{ m}^3
\]
Deformation of Solids

Post and Beam Arches

- A horizontal beam is supported by two columns
- Used in Greek temples
- Columns are closely spaced
  - Limited length of available stones
  - Low ultimate tensile strength of sagging stone beams
Deformation of Solids

Semicircular Arch

- Developed by the Romans
- Allows a wide roof span on narrow supporting columns
- Stability depends upon the compression of the wedge-shaped stones
Deformation of Solids

Gothic Arch

- First used in Europe in the 12\textsuperscript{th} century
- Extremely high
- The \textit{flying buttresses} are needed to prevent the spreading of the arch supported by the tall, narrow columns
Density & Pressure

Density

- The density of a substance of uniform composition is defined as its mass per unit volume:

\[ \rho = \frac{m}{V} \]

- Units are kg/m\(^3\) (SI) or g/cm\(^3\) (cgs)
- 1 g/cm\(^3\) = 1000 kg/m\(^3\)
The densities of most **liquids and solids** vary **slightly** with changes in temperature and pressure

- Densities of **gases** vary **greatly** with changes in temperature and pressure
Density & Pressure

Specific Gravity

- The *specific gravity* of a substance is the ratio of its density to the *density of water* at 4°C
  - The density of water at 4°C is **1000 kg/m³**
- Specific gravity is a unitless ratio
Density & Pressure

Pressure

- The force exerted by a fluid on a submerged object at any point if perpendicular to the surface of the object

\[ P = \frac{F}{A} \text{ in } Pa = \frac{N}{m^2} \]
Density & Pressure

Measuring Pressure

- The spring is calibrated by a known force
- The force the fluid exerts on the piston is then measured
Variation of Pressure with Depth

- If a fluid is at rest in a container, all portions of the fluid must be in static equilibrium.
- All points at the same depth must be at the same pressure:
  - Otherwise, the fluid would not be in equilibrium.
  - The fluid would flow from the higher pressure region to the lower pressure region.
Variation of Pressure with Depth

- Examine the darker region, assumed to be a fluid
  - It has a cross-sectional area $A$
  - Extends to a depth $h$ below the surface
- Three external forces act on the region
Variation of Pressure with Depth

- \( P = P_0 + \rho gh \)
- \( P_0 \) is normal atmospheric pressure
  - \( 1.013 \times 10^5 \) Pa = 14.7 lb/in\(^2\)
- The pressure does not depend upon the shape of the container
Example 3

Problem In a huge oil tanker, salt water has flooded an oil tank to a depth of 5.00 m. On top of the water is a layer of oil 8.00 m deep, as in the cross-sectional view of the tank in Figure. The oil has a density of 0.700 g/cm³. Find the pressure at the bottom of the tank. (Take 1.025 kg/m³ as the density of salt water.)

\[ P_1 = P_0 + \rho g h_1 \]
\[ = 1.01 \times 10^5 \text{ Pa} \]
\[ + (7.00 \times 10^2 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(8.00 \text{ m}) \]
\[ P_1 = 1.56 \times 10^5 \text{ Pa} \]

\[ P_{\text{bot}} = P_1 + \rho g h_2 \]
\[ = 1.56 \times 10^5 \text{ Pa} \]
\[ + (1.025 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(5.00 \text{ m}) \]
\[ P_{\text{bot}} = 2.06 \times 10^5 \text{ Pa} \]
Variation of Pressure with Depth

Pascal’s Principle

- A change in pressure applied to an enclosed fluid is transmitted undiminished to every point of the fluid and to the walls of the container.
  - First recognized by Blaise Pascal, a French scientist (1623 – 1662)
Variation of Pressure with Depth

- The hydraulic press is an important application of Pascal’s Principle

\[ P = \frac{F_1}{A_1} = \frac{F_2}{A_2} \]

- Also used in hydraulic brakes, forklifts, car lifts, etc.
Example 4

Problem In a car lift used in a service station, compressed air exerts a force on a small piston of circular cross section having a radius of \( r_1 = 5.00 \text{ cm} \). This pressure is transmitted by an incompressible liquid to a second piston of radius \( r_2 = 15.0 \text{ cm} \). (a) What force must the compressed air exert on the small piston in order to lift a car weighing 13 300 N? Neglect the weights of the pistons. (b) What air pressure will produce a force of that magnitude? (c) Show that the work done by the input and output pistons is the same.

\[
F_1 = \left( \frac{A_1}{A_2} \right) F_2 = \frac{\pi r_1^2}{\pi r_2^2} F_2 \\
= \frac{\pi (5.00 \times 10^{-2} \text{ m})^2}{\pi (15.0 \times 10^{-2} \text{ m})^2} (1.33 \times 10^4 \text{ N}) \\
= 1.48 \times 10^3 \text{ N}
\]

\[
P = \frac{F_1}{A_1} = \frac{1.48 \times 10^3 \text{ N}}{\pi (5.00 \times 10^{-2} \text{ m})^2} = 1.88 \times 10^5 \text{ Pa}
\]

\[
V_1 = V_2 \implies A_1 \Delta x_1 = A_2 \Delta x_2
\]

\[
\frac{A_2}{A_1} = \frac{\Delta x_1}{\Delta x_2}
\]

\[
\frac{F_1}{A_1} = \frac{F_2}{A_2} \implies \frac{F_1}{F_2} = \frac{A_1}{A_2}
\]

\[
\frac{W_1}{W_2} = \frac{F_1 \Delta x_1}{F_2 \Delta x_2} = \left( \frac{F_1}{F_2} \right) \left( \frac{\Delta x_1}{\Delta x_2} \right) = \left( \frac{A_1}{A_2} \right) \left( \frac{A_2}{A_1} \right) = 1
\]

\[
W_1 = W_2
\]
Variation of Pressure with Depth

Absolute vs. Gauge Pressure

- The pressure $P$ is called the *absolute* pressure
  - Remember, $P = P_0 + \rho gh$

- $P - P_0 = \rho gh$ is the *gauge* pressure
Pressure Measurements

Manometer 壓力計

- One end of the U-shaped tube is open to the atmosphere
- The other end is connected to the pressure to be measured
- Pressure at B is $P_{o} + \rho gh$
Pressure Measurements

Blood pressure

- Blood pressure is measured with a special type of manometer called a sphygmomanometer.
- Pressure is measured in mm of mercury.
Pressure Measurements

Barometer 氣壓計

- Invented by Torricelli (1608 – 1647)
- A long closed tube is filled with mercury and inverted in a dish of mercury
- Measures atmospheric pressure as $\rho gh$

$P = 0$

$h$

$P_0$
Pressure Measurements

Pressure Values in Various Units

- One atmosphere of pressure is defined as the pressure equivalent to a column of mercury exactly 0.76 m tall at 0º C where g = 9.81 m/s²

- One atmosphere (1 atm) =
  - 76.0 cm of mercury
  - 1.013 x 10⁵ Pa
  - 14.7 lb/in²
Buoyant Forces & Archimedes’s Principle

Archimedes

- 287 – 212 BC
- Greek mathematician, physicist, and engineer
- Buoyant force
- Inventor
Archimedes' Principle

- Any object completely or partially submerged in a fluid is **buoyed up by a force** whose magnitude is equal to the **weight of the fluid displaced** by the object.
Buoyant Forces & Archimedes’s Principle

Buoyant Force

- The upward force is called the **buoyant force**
- The physical cause of the buoyant force is the **pressure difference** between the top and the bottom of the object
Buoyant Forces & Archimedes’s Principle

- The magnitude of the buoyant force always equals the weight of the displaced fluid
  \[ B = \rho_{\text{fluid}} V_{\text{fluid}} g = w_{\text{fluid}} \]

- The buoyant force is the same for a **totally submerged** object of any size, shape, or density

- The buoyant force is exerted by the fluid

- Whether an object **sinks** or **floats** depends on the relationship between the buoyant force and the weight
Buoyant Forces & Archimedes’s Principle

Totally Submerged Object

- The upward buoyant force is \( B = \rho_{\text{fluid}} g V_{\text{obj}} \)

- The downward gravitational force is \( w = mg = \rho_{\text{obj}} g V_{\text{obj}} \)

- The net force is \( B - w = (\rho_{\text{fluid}} - \rho_{\text{obj}}) g V_{\text{obj}} \)
Buoyant Forces & Archimedes’s Principle

- The object is less dense than the fluid
- The object experiences a net upward force
The object is more dense than the fluid
The net force is downward
The object accelerates downward
Example 5

**Problem** A bargain hunter purchases a “gold” crown at a flea market. After she gets home, she hangs it from a scale and finds its weight to be 7.84 N. She then weighs the crown while it is immersed in water, and now the scale reads 6.86 N. Is the crown made of pure gold?

\[
T_{\text{air}} - mg = 0
\]

\[
T_{\text{water}} - mg + B = 0
\]

\[
B = T_{\text{air}} - T_{\text{water}} = 7.84 \text{ N} - 6.86 \text{ N} = 0.980 \text{ N}
\]

\[
B = \rho_{\text{water}} g V_{\text{water}} = 0.980 \text{ N}
\]

\[
V_{\text{water}} = \frac{0.980 \text{ N}}{g \rho_{\text{water}}} = \frac{0.980 \text{ N}}{(9.80 \text{ m/s}^2)(1.00 \times 10^3 \text{ kg/m}^3)} - 1.00 \times 10^{-4} \text{ m}^3
\]

\[
m = \frac{T_{\text{air}}}{g} = \frac{7.84 \text{ N}}{9.80 \text{ m/s}^2} = 0.800 \text{ kg}
\]

\[
\rho_{\text{crown}} = \frac{m}{V_{\text{crown}}} = \frac{0.800 \text{ kg}}{1.00 \times 10^{-4} \text{ m}^3} = 8.00 \times 10^3 \text{ kg/m}^3
\]

Because the density of gold is $19.3 \times 10^3 \text{ kg/m}^3$, the crown is either hollow, made of an alloy, or both.
Buoyant Forces & Archimedes’s Principle

Floating Object

- The object is in **static equilibrium**
- The upward buoyant force is balanced by the downward force of gravity
- Volume of the fluid displaced corresponds to the volume of the object beneath the fluid level
Buoyant Forces & Archimedes’s Principle

- **The forces balance**

\[
\frac{\rho_{\text{obj}}}{\rho_{\text{fluid}}} = \frac{V_{\text{fluid}}}{V_{\text{obj}}}
\]
Example 6

Problem  A raft is constructed of wood having a density of $6.00 \times 10^2$ kg/m$^3$. Its surface area is $5.70$ m$^2$, and its volume is $0.60$ m$^3$. When the raft is placed in fresh water as in Figure, to what depth $h$ is the bottom of the raft submerged?

\[
B - m_{\text{raft}}g = 0 \quad \rightarrow \quad B = m_{\text{raft}}g
\]

\[
B = m_{\text{water}}g = (\rho_{\text{water}}V_{\text{water}})g = (\rho_{\text{water}}Ah)g
\]

\[
m_{\text{raft}}g = (\rho_{\text{raft}}V_{\text{raft}})g
\]

\[
(\rho_{\text{water}}Ah)g = (\rho_{\text{raft}}V_{\text{raft}})g
\]

\[
h = \frac{\rho_{\text{raft}}V_{\text{raft}}}{\rho_{\text{water}}A}
\]

\[
= \frac{(6.00 \times 10^2 \text{ kg/m}^3)(0.600 \text{ m}^3)}{(1.00 \times 10^3 \text{ kg/m}^3)(5.70 \text{ m}^2)}
\]

\[
= 0.063 \text{ m}
\]
Fluids in Motion

Streamline Flow 流線流

- **Streamline flow**
  - Every particle that passes a particular point moves exactly along the smooth path followed by particles that passed the point earlier
  - Also called *laminar flow* 層流

- **Streamline is the path**
  - Different streamlines cannot cross each other
  - The streamline at any point coincides with the direction of fluid velocity at that point
Fluids in Motion

Streamline flow shown around an auto in a wind tunnel
Turbulent Flow 紊流、湍流、乱流

- The flow becomes irregular
  - exceeds a certain velocity
  - any condition that causes abrupt changes in velocity
- Eddy currents 渦流 are a characteristic of turbulent flow
Fluids in Motion

Viscosity

- **Viscosity** is the degree of internal friction in the fluid
- The internal friction is associated with the resistance between two adjacent layers of the fluid moving relative to each other
Fluids in Motion

Characteristics of an Ideal Fluid

- **The fluid is nonviscous**
  - There is no internal friction between adjacent layers
- **The fluid is incompressible**
  - Its density is constant
- **The fluid motion is steady**
  - Its velocity, density, and pressure do not change in time
- **The fluid moves without turbulence**
  - No eddy currents are present
  - The elements have zero angular velocity about its center
Fluids in Motion

Equation of Continuity

- $A_1 v_1 = A_2 v_2$
- The product of the cross-sectional area of a pipe and the fluid speed is a constant
  - Speed is high where the pipe is narrow and speed is low where the pipe has a large diameter
- $A_v$ is called the flow rate
Fluids in Motion

- The equation is a consequence of conservation of mass and a steady flow.
- A v = constant
  - This is equivalent to the fact that the volume of fluid that enters one end of the tube in a given time interval equals the volume of fluid leaving the tube in the same interval.
  - Assumes the fluid is incompressible and there are no leaks.
**Example 7**

**Problem** A water hose 2.50 cm in diameter is used by a gardener to fill a 30.0-liter bucket. (One liter = 1000 cm³.) The gardener notices that it takes 1.00 min to fill the bucket. A nozzle with an opening of cross-sectional area 0.500 cm² is then attached to the hose. The nozzle is held so that water is projected horizontally from a point 1.00 m above the ground. Over what horizontal distance can the water be projected?

\[
\text{volume flow rate} = \frac{30.0 \text{ L}}{1.00 \text{ min}} \left( \frac{1.00 \times 10^3 \text{ cm}^3}{1.00 \text{ L}} \right) \left( \frac{1.00 \text{ m}}{100.0 \text{ cm}} \right)^3 \left( \frac{1.00 \text{ min}}{60.0 \text{ s}} \right) = 5.00 \times 10^{-4} \text{ m}^3/\text{s}
\]

\[
A_1 v_1 = A_2 v_2 - A_2 v_{0x}
\]

\[
v_{0x} = \frac{A_1 v_1}{A_2} = \frac{5.00 \times 10^{-4} \text{ m}^3/\text{s}}{0.500 \times 10^{-4} \text{ m}^2} = 10.0 \text{ m/s}
\]

\[
\Delta y = v_{0y} t - \frac{1}{2} gt^2
\]

\[
t = \sqrt{\frac{-2\Delta y}{g}} = \sqrt{\frac{-2(-1.00 \text{ m})}{9.80 \text{ m/s}^2}} = 0.452 \text{ s}
\]

\[
x = v_{0x} t = (10.0 \text{ m/s})(0.452 \text{ s}) = 4.52 \text{ m}
\]
Fluids in Motion

Daniel Bernoulli

- 1700 – 1782
- Swiss physicist and mathematician
- Wrote *Hydrodynamica*
- Also did work that was the beginning of the kinetic theory of gases
Fluids in Motion

Bernoulli’s Equation

- Relates pressure to fluid speed and elevation
- Bernoulli’s equation is a consequence of Conservation of Energy applied to an ideal fluid
- Assumes the fluid is incompressible and nonviscous, and flows in a nonturbulent, steady-state manner
- States that the sum of the pressure, kinetic energy per unit volume, and the potential energy per unit volume has the same value at all points along a streamline

\[
P + \frac{1}{2} \rho v^2 + \rho gy = \text{constant}
\]
Fluids in Motion

Venturi Tube  文氏管

- Shows fluid flowing through a horizontal constricted pipe
- Speed changes as diameter changes
- Can be used to measure the speed of the fluid flow
- Swiftly moving fluids exert less pressure than do slowly moving fluids
**Example 8**

**Problem** A nearsighted sheriff fires at a cattle rustler with his trusty six-shooter. Fortunately for the rustler, the bullet misses him and penetrates the town water tank, causing a leak. If the top of the tank is open to the atmosphere, determine the speed at which the water leaves the hole when the water level is 0.500 m above the hole. (b) Where does the stream hit the ground if the hole is 3.00 m above the ground?

\[
P_0 + \frac{1}{2} \rho v_1^2 + \rho g y_1 = P_0 + \rho g y_2
\]

\[
v_1 = \sqrt{2g(y_2 - y_1)} = \sqrt{2gh}
\]

\[
v_1 = \sqrt{2(9.80 \text{ m/s}^2)(0.500 \text{ m})} = 3.13 \text{ m/s}
\]

\[
\Delta y = -\frac{1}{2}gt^2 + v_{0y}t
\]

\[-3.00 \text{ m} - (-4.90 \text{ m/s}^2)t^2
\]

\[t = 0.782 \text{ s}
\]

\[
x = v_{0x}t = (3.13 \text{ m/s})(0.782 \text{ s}) = 2.45 \text{ m}
\]
Fluids in Motion

An Object Moving Through a Fluid

- Many common phenomena can be explained by Bernoulli’s equation
  - At least partially
- In general, an object moving through a fluid is acted upon by a net upward force as the result of any effect that causes the fluid to change its direction as it flows past the object
Applications of Fluid Dynamics

- The dimples in the golf ball help move air along its surface.
- The ball pushes the air down.
- Newton’s Third Law tells us the air must push up on the ball.
- The spinning ball travels farther than if it were not spinning.
Applications of Fluid Dynamics

- The air speed above the wing is greater than the speed below
- The air pressure above the wing is less than the air pressure below
- There is a net upward force
  - Called lift
- Other factors are also involved
Surface Tension, Capillary Action, & Viscous Fluid Flow

Surface Tension

- **Net force on molecule A is zero**
  - Pulled equally in all directions

- **Net force on B is not zero**
  - No molecules above to act on it
  - Pulled toward the center of the fluid
The net effect of this pull on all the surface molecules is to make the surface of the liquid contract.

- Makes the surface area of the liquid as small as possible.
  - Example: Water droplets take on a spherical shape since a sphere has the smallest surface area for a given volume.
Surface Tension, Capillary Action, & Viscous Fluid Flow

Surface Tension on a Needle

- Surface tension allows the needle to float, even though the density of the steel in the needle is much higher than the density of the water.
- The needle actually rests in a small depression in the liquid surface.
- The vertical components of the force balance the weight.
Surface Tension, Capillary Action, & Viscous Fluid Flow

Surface Tension, Equation

- The surface tension is defined as the ratio of the magnitude of the surface tension force to the length along which the force acts:

\[ \sigma = \frac{F}{L} \]

- SI units are N/m
- In terms of energy, any equilibrium configuration of an object is one in which the energy is a minimum
Surface Tension, Capillary Action, & Viscous Fluid Flow

Measuring Surface Tension

- The force is measured just as the ring breaks free from the film

\[ \sigma = \frac{F}{2L} \]

- The 2L is due to the force being exerted on the inside and outside of the ring
The surface tension of liquids decreases with increasing temperature.

Surface tension can be decreased by adding ingredients called *surfactants* to a liquid.

- Detergent is an example.
Surface Tension, Capillary Action, & Viscous Fluid Flow

- **Cohesive forces** are forces between **like** molecules
- **Adhesive forces** are forces between **unlike** molecules
- The shape of the surface depends upon the relative size of the cohesive and adhesive forces
The adhesive forces are greater than the cohesive forces.

The liquid clings to the walls of the container.

The liquid “wets” the surface.
Liquids in Contact with a Solid Surface – Case 2

- Cohesive forces are greater than the adhesive forces
- The liquid curves downward
- The liquid does not “wet” the surface
Surface Tension, Capillary Action, & Viscous Fluid Flow

Contact Angle

- In a, $\Phi > 90^\circ$ and cohesive forces are greater than adhesive forces
- In b, $\Phi < 90^\circ$ and adhesive forces are greater than cohesive forces
Capillary Action

- Capillary action is the result of surface tension and adhesive forces.
- The liquid rises in the tube when adhesive forces are greater than cohesive forces.
- At the point of contact between the liquid and the solid, the upward forces are as shown in the diagram.
Surface Tension, Capillary Action, & Viscous Fluid Flow

- Here, the cohesive forces are greater than the adhesive forces.
- The level of the fluid in the tube will be below the surface of the surrounding fluid.
The height at which the fluid is drawn above or depressed below the surface of the surrounding liquid is given by:

\[ h = \frac{2\sigma}{\rho gr} \cos \phi \]
Viscous Fluid Flow

- Viscosity refers to friction between the layers
- Layers in a viscous fluid have different velocities
- The velocity is greatest at the center
- Adhesive forces between the fluid and the walls slow down the fluid on the outside
Surface Tension, Capillary Action, & Viscous Fluid Flow

Coefficient of Viscosity

- Assume a fluid between two solid surfaces
- A force is required to move the upper surface

\[ F = \eta \frac{Av}{d} \]

- \( \eta \) is the coefficient
- \( A \) is area
- SI units are N \( \cdot \) s/m²
- cgs units are Poise
  - 1 Poise = 0.1 N \( \cdot \) s/m²
# Surface Tension, Capillary Action, & Viscous Fluid Flow

<table>
<thead>
<tr>
<th>Fluid</th>
<th>$T$ (°C)</th>
<th>Viscosity $\eta$ (N⋅s/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>20</td>
<td>$1.0 \times 10^{-3}$</td>
</tr>
<tr>
<td>Water</td>
<td>100</td>
<td>$0.3 \times 10^{-3}$</td>
</tr>
<tr>
<td>Whole blood</td>
<td>37</td>
<td>$2.7 \times 10^{-3}$</td>
</tr>
<tr>
<td>Glycerin</td>
<td>20</td>
<td>$1500 \times 10^{-3}$</td>
</tr>
<tr>
<td>10-wt motor oil</td>
<td>30</td>
<td>$250 \times 10^{-3}$</td>
</tr>
</tbody>
</table>
Poiseuille’s Law

- Gives the *rate of flow* of a fluid in a tube with pressure differences

\[
\text{Rate of flow} = \frac{\Delta V}{\Delta t} = \frac{\pi R^4 (P_1 - P_2)}{8 \eta L}
\]
Example 9

**Problem**  A patient receives a blood transfusion through a needle of radius 0.20 mm and length 2.0 cm. The density of blood is 1.050 kg/m³. The bottle supplying the blood is 0.50 m above the patient’s arm. What is the rate of flow through the needle?

\[
P_1 - P_2 = \rho gh = (1.050 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(0.50 \text{ m})
\]
\[
= 5.15 \times 10^3 \text{ Pa}
\]

\[
\frac{\Delta V}{\Delta t} = \frac{\pi R^4 (P_1 - P_2)}{8 \eta L}
\]
\[
= \frac{\pi (2.0 \times 10^{-4} \text{ m})^4 (5.15 \times 10^3 \text{ Pa})}{8 (2.7 \times 10^{-3} \text{ N} \cdot \text{s/m}^2) (2.0 \times 10^{-2} \text{ m})}
\]
\[
= 6.0 \times 10^{-8} \text{ m}^3/\text{s}
\]
Reynold’s Number

At sufficiently high velocity, a fluid flow can change from streamline to turbulent flow

- The onset of turbulence can be found by a factor called the Reynold’s Number, $R_N$

$$R_N = \frac{\rho v d}{\eta}$$

- $v$: mean velocity
- $d$: tube/capillary diameter
- $\rho$: density of liquid
- $\eta$: coefficient of viscosity

- If $R_N = 2000$ or below, flow is streamline
- If $2000 < R_N < 3000$, the flow is unstable
- If $R_N = 3000$ or above, the flow is turbulent
Transport Phenomena

- Movement of a fluid may be due to differences in concentration
  - As opposed to movement due to a pressure difference
  - Concentration is the number of molecules per unit volume
- The fluid will flow from an area of high concentration to an area of low concentration
- The processes are called diffusion and osmosis.

Transport Phenomena

Diffusion and Fick’s Law

- Molecules move from a region of high concentration to a region of low concentration
- Basic equation for diffusion is given by Fick’s Law

\[
\text{Diffusion rate} = \frac{\text{Mass}}{\text{time}} = D A \left(\frac{C_2 - C_1}{L}\right)
\]

- \(D\) is the diffusion coefficient
Transport Phenomena

Diffusion 擴散

- Concentration on the left is higher than on the right of the imaginary barrier
- Many of the molecules on the left can pass to the right, but few can pass from right to left
- There is a net movement from the higher concentration to the lower concentration
Osmosis is the movement of water from a region where its concentration is high, across a selectively permeable membrane, into a region where its concentration is lower.

- A *selectively permeable membrane* is one that allows passage of some molecules, but not others.

http://www.tvdsb.on.ca/westmin/science/Sbi3a1/cells/Osmosis.htm
Transport Phenomena

Motion Through a Viscous Medium

- When an object falls through a fluid, a viscous drag acts on it
- The resistive force on a small, spherical object of radius \( r \) falling through a viscous fluid is given by *Stoke’s Law*:

\[
F_r = 6 \pi \eta r v
\]
Motion in a Viscous Medium

- As the object falls, three forces act on the object.
- As its speed increases, so does the resistive force.
- At a particular speed, called the *terminal speed*, the net force is zero.

\[ v_t = \frac{2r^2g}{9\eta} (\rho - \rho_f) \]
Transport Phenomena

Terminal Velocity

- Stokes’ Law will not work if the object is not spherical
- Assume the resistive force has a magnitude given by $F_r = k \cdot v$
  - $k$ is a coefficient to be determined experimentally
- The terminal velocity will become

$$v_t = \frac{m g}{k} \left(1 - \frac{\rho_f}{\rho}\right)$$
Transport Phenomena

Sedimentation Rate

- The speed at which materials fall through a fluid is called the sedimentation rate
  - It is important in clinical analysis
- The rate can be increased by increasing the effective value of g
  - This can be done in a centrifuge
Transport Phenomena

Centrifuge

- High angular speeds give the particles a large radial acceleration
  - Much greater than $g$
  - In the equation, $g$ is replaced with $w^2r$
The particles’ terminal velocity will become

\[ v_t = \frac{m \omega^2 r}{k} \left(1 - \frac{\rho_f}{\rho}\right) \]

The particles with greatest mass will have the greatest terminal velocity

The most massive particles will settle out on the bottom of the test tube first
1. Styrofoam has a density of 150 kg/m³. What is the maximum mass that can hang without sinking from a 50-cm-diameter Styrofoam sphere in water? Assume the volume of the mass is negligible compared to that of the sphere.

2. What does the top pressure gauge read?

3. a. The 70 kg student in the figure balances a 1200 kg elephant on a hydraulic lift. What is the diameter of the piston the student is standing on?
   b. A second 70 kg student joins the first student. How high do they lift the elephant?
Assignment 10

4. Air flows through this tube at a rate of $1200 \text{ cm}^3/\text{s}$. Assume that air is an ideal fluid. What is the height $h$ of mercury in the right side of the U-tube?

5. A 4.0-mm-diameter hole is 1.0 m below the surface of a 2.0-m-diameter tank of water.
   a. What is the volume flow rate through the hole, in liter/min?
   b. What is the rate, in mm/min, at which the water level in the tank will drop if the water is not replenished?