Abstract

This research aims to investigate the bearing capacity of shallow foundation on the granular materials. The software PFC2D based on the distinct element method is adopted. The parameters including the packing of particles, the basic friction angle, the particle size and the particle stiffness are considered to find which the major influence factor of the bearing capacity is. The simulation results show that: (1) in the dense packing state, the failure pattern is similar to the general shear failure, and in the loose packing state, the pattern reflects the punch failure; (2) an increasing friction angle increase the bearing capacity; (3) the larger size and the higher stiffness will lead to the higher strength, but these major influence factors are not considered in the previous limit equilibrium theory.

Keywords: shallow foundation, distinct element method, granular material.

1 Introduction

Foundation means something that supports the upper structure along with the external loads, and it is usually divided into two categories: shallow foundation and deep foundation. For shallow foundation, it mainly transfers external loads to relatively small depths into the ground. In order to fulfil the design purpose, i.e. the bearing capacity, shallow foundation has to safe against overall shear failure in the soil. Traditionally, the bearing capacity theory of shallow foundation is obtained from limit equilibrium method as suggested by Terzaghi [1]. This theory has some features, such as simple, reasonably good accuracy, that appeal to geotechnical engineers. However, for granular material, such as sand and gravel, its behaviour is significant influenced by the particle features, which are difficult to model from the continuum mechanics. Therefore, the particle-based method provides an alternative to the aforementioned issue.
This research aims to investigate the bearing capacity of shallow foundation through the distinct element method. In order to understand the relations between the properties of particle bearing capacity, the software PFC2D, presented by Cundall and Strack [2], is adopted. The parameters including the packing of particle, the basic friction angle, the particle size and the particle stiffness are considered to find which the major influence factor of the bearing capacity is.

2 Bearing capacity of shallow foundation by limit equilibrium method

Usually, the bearing capacity of a foundation is determined by limit equilibrium, or limit analysis methods. In the following, the basic theory of limit equilibrium will be briefly described.

For a rigid footing punching in a homogeneous soil under half space, Prandtl [3] considered that the wedge failure mechanism occurs. Based on the assumptions: (1) The soil is homogeneous, isotropic and weightless ($\gamma = 0$); (2) the bottom surface of the footing is smooth with no friction; (3) the footing is subjected to vertical center load, and the plastic theory, Prandtl proposed the ultimate bearing capacity of the footing as:

$$q_u = \frac{c}{\tan \phi} \left[ \tan^2 \left( \frac{\pi}{4} + \frac{\phi}{2} \right) e^{\phi \tan \phi} - 1 \right]$$

(1)

Where $q_u$ is ultimate bearing capacity; $c$ is cohesion of soil; $\phi$ is friction angle.

Terzaghi [1] extended Prandtl’s plastic theory with more consideration, which includes: (1) the soil is a homogeneous, isotropic, rigid-plastic material; (2) the bottom surface of the footing is rough; (3) general shear failure occurs; (4) the failure zone includes Rankine active zone, Rankine passive zone, and transition area; (5) the shear strength of the soil above the footing base is negligible; and (6) the soil above the footing base can be replaced by an equivalent surcharge ($q = \gamma D$), and his failure mechanism is shown as Fig. 1.

Therefore, the well-known Terzaghi’s bearing capacity theory, based on equilibrium analysis, could be expressed as:

$$q_u = cN_c + qN_q + \frac{1}{2} \gamma BN_y$$

(2)

Where $N_c, N_q, N_y$ are bearing capacity factors that are non-dimensional and are functions only of the soil friction angle. The bearing capacity factors are defined by

$$N_c = (N_q - 1) \cot \phi$$

(3)
\[ N_q = \frac{1}{2} e^{2 \tan \phi} \tan^2 \left( 45^\circ + \frac{\phi}{2} \right) \]  

(4)

\[ N_p = \frac{1}{2} \tan \phi \left( \frac{K_{py}}{\cos^2 \phi} - 1 \right) \]  

(5)

Where \( k_{py} \) is passive pressure coefficient.

![Figure 1 Bearing capacity analysis proposed by Terzaghi [1].](image)

Furthermore, based on Terzaghi’s equation, numerous researchers have developed modifications for different practical situations [4-6]. These limit equilibrium methods of determining bearing capacity have proven useful for practical foundation engineering. Therefore, the simulated results in this research will compare with the Terzaghi’s theory in the following section.

3 Methodology

3.1 Description of distinct-element method

In order to simulate the deformation characteristics of granular material, the software PFC2D based on the distinct element method (DEM) is adopted in this research. The DEM was firstly proposed by Cundall [7]. In the calculation process, the translational and rotational motion of each particle is determined by Newton’s Second Law, while the force-displacement law is used to analyze the contact forces from the relative motion at each contact.

In PFC2D, The basic elements are viewed as circular or spherical rigid particles with a finite mass [8]. When one particle contacts with another one or boundary, the contact force is formed and it can be divided into normal and shear components with respect to the contact plane. The normal force can be expressed as
\[ F_i^n = K^n U^n \]  \hspace{1cm} (6)

where \( U^n \) is the overlap and \( K^n \) is the contact normal stiffness.

Similarly, the shear force increment is given by

\[ \Delta F_i^s = -k^s \Delta U_i^s \]  \hspace{1cm} (7)

where \( k^s \) is the contact shear stiffness.

Furthermore, the slip between two particles is allowed to occur when the maximum shear force equals the friction (\( F^s = \mu F^n \), \( \mu \) is the friction coefficient).

### 3.2 Model generation

This research aims to explore the influence of different geometry and mechanical properties of particle on bearing capacity of shallow foundation. The bearing capacity of strip foundation is a typical plane strain problem. In order to compare with the following experiments, a sandbox model with 600 mm in width and 300 mm in height was analysed as shown in Fig. 2. The lateral and lower boundaries were considered as rigid plates, and the upper boundary was unconstrained. A rigid frictional foundation with 42 mm in width was put on the cohesionless particles. Then, the compression test was simulated by vertically moving the foundation downward at a constant velocity of 0.1 mm/s.

![Figure 2 PFC2D model used for this study.](image)

### 3.3 Geometric parameters and packing of particles

According to Vesic’s [6] research, the modes of foundation failure, including general shear failure, local shear failure and punching failure, are related to the relative density (\( Dr \)) of particles. For higher relative density, the general shear
failure occurs. In order to investigate the influence of relative density, two extreme
conditions with equal-sized spheres, simple cubic packing (loose packing) and dense
packing as shown in Fig. 3, were considered in this research. The void ratio of two
packing types is 0.91 and 0.65, respectively. In addition, four cases of particle size
were chosen as 1mm, 2mm, 4mm and 8mm.

![Schematic diagram of particle packing.](image)

(a) Loose packing  (b) Dense packing

3.4 Mechanical properties of particle

In this research, linear contact model with no bonding was used as the inter-particle
model to describe the constitutive behaviour between two particles. Therefore, the
micro-properties \{K_n, k_s, \mu\} and the density need to be determined. For granular
material of following tests, the spherical quartz particle will be selected. Usually, the
micro-parameters are determined through a calibration process by matching the
observed macroscopic behaviour with the simulation. However, these micro-
properties here are straight determined based on the natural properties of quartz, and
the procedure is described as follows:

(A) The density of quartz is 2650 kg/cm³;
(B) The Young’s modulus of quartz is $7.3 \times 10^7$ kPa, and its Poisson’s ratio is
0.17. Then, based on Wang et al. [9], the normal and shear stiffness were
calculated as

\[
K_n = \frac{2 \sqrt{3} E t}{3(1-\nu)}
\]

(8)

\[
\frac{k_s}{K_n} = \frac{1-3\nu}{1+\nu}
\]

(9)

In addition, in order to explore the influence of stiffness, a higher stiffness
and a lower stiffness of particle are simulated in this research.
(C) The friction coefficient here means the basic friction of particle. In order to
understand the influence of basic friction angle on bearing capacity, the
friction angle was chosen from 5° to 35°, which friction coefficient ranges from 0.087 to 0.700.

Based on aforementioned properties, all parameters needed in DEM model are shown in Table 1.

<table>
<thead>
<tr>
<th>Items</th>
<th>Values (units)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>2.65×10³ (kg/m³)</td>
</tr>
<tr>
<td>Normal stiffness</td>
<td>1×10⁴; 1×10⁶; 1×10⁸ (kN/m)</td>
</tr>
<tr>
<td>Shear stiffness</td>
<td>4.14×10³; 4.14×10⁵; 4.14×10⁷ (kN/m)</td>
</tr>
<tr>
<td>Friction coefficient</td>
<td>0.087 – 0.700</td>
</tr>
</tbody>
</table>

Table 1 Microproperties of particles

4 Results and discussion

According to the aforementioned model, the influence of friction angle, packing type, particle size, and stiffness on bearing capacity and failure pattern of foundation is discussed in this section. Moreover, the results are compared with the limit equilibrium analysis.

4.1 Failure pattern

Fig. 4 illustrates the displacement patterns of dense packing at failure, and the general shear failure mode can be observed. These patterns are similar to that of plastic theory, as shown in Fig. 1, and the failure zone under the foundation can be also separated into three zones: the triangular active zone; the transition zone and the passive zone. Furthermore, two types of general shear failure pattern can be identified, Type I and Type II, as shown in Fig. 4a and 4b, respectively. Compared with Type I, Type II exhibits similar active and transition zone but much larger passive zone, which depth is approximately three times more than that of transition area. It is found that Type II failure occurs due to the higher friction angle. Moreover, the simulated shear zones with different friction angles are presented in Fig. 5, and the result of limit equilibrium analysis based on $\phi = 35^0$ is also demonstrated. It reveals that the shear zone extends as the friction angle increases. Compared with the shear zone form plastic theory ($\phi = 35^0$), the simulated shear zone is much larger, especially on the depth.

On the other hand, Fig. 6 exhibits the displacement pattern of simple cubic packing (loose packing) at failure. A punching failure could be observed, and only a narrow shear zone developed. This phenomenon coincides with the conclusion of Vesic’s [6] research.
Figure 4  The failure mechanisms of dense packing.

(a) Type I ($\phi = 20^\circ$)                             (b) Type II ($\phi = 25^\circ$)

Figure 5  The failure patterns of different friction angles under dense packing.
4.2 Bearing capacity by DEM

(1) Influence of friction angle

Fig. 7 depicts the variations of bearing capacity of two packing types with different friction angles, ranging from 5° to 35°. It is clear that the strength of the dense packing is significantly greater than that of the loose packing, and the strength of two packing type increases as the angle arises. Furthermore, the strength curve of dense packing can be regarded as an upper limit of bearing capacity. On the contrary, the curve of loose packing as a lower limit, and the zone between the two curves reflect the strength owing to the different particle packing types.

In addition, the result based on Terzaghi’s bearing capacity is also demonstrated in Fig. 7. However, keeping in mind that the friction angle of limit equilibrium method means the apparent angle, not the basic friction angle. It could be found that Terzaghi’s bearing capacity is less than that of both the loose and dense packing when the friction angle is less than 35°. As the angle is more than 30°, Terzaghi’s strength is between the two limits. The analysis results indicate that the tendencies of two methods are similar, but the actual values are apparently different. Terzaghi’s bearing capacity seems to be a more conservative analysis.
(2) Influence of particle size

Figure 8 illustrates the simulated strength versus different particle sizes. Under either dense packing or loose packing, the results reveal that the strength increase as the particle size increases. Especially for dense packing, the strength is significantly influenced by the particle size. For instance, the bearing capacity of 8mm in particle size is almost four times than that of 1mm in size. Therefore, the bearing capacity of granular materials is affected by the particle size.

(3) Influence of stiffness

Traditionally, the bearing capacity of plastic theory depends only on the friction angle, the cohesion, and the density of soil, and it gives no consideration to the
deformation modulus of soil. However, in this research, it is found that the strength of footings is significantly affected by the stiffness of particle, as shown in Fig. 9, and the strength increases apparently with the particle stiffness. Compared with the influence of the packing, or the friction angle, the particle stiffness seems to be one of major factors influencing the bearing capacity.

Figure 9  Variation of strength with different particle stiffness.

5  Conclusion

This research explores the influence factors, including the packing, the basic friction angle, the particle size and the particle stiffness, on the bearing capacity of shallow foundation. The software PFC2D based on the distinct element method is adopted, and the simulation results show that: (1) in the dense packing state, the failure pattern is similar to the general shear failure, and in the loose packing state, the pattern reflects the punch failure; (2) an increasing friction angle increase the bearing capacity; (3) the larger size and the higher stiffness will lead to the higher strength, but these major influence factors are not considered in the previous limit equilibrium theory.

Acknowledgements

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References